Which cosmological model with dark energy — phantom or ΛCDM?

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In cosmology many dramatically different scenarios with the past (big bang versus bounce) and in the future (de Sitter versus big rip) singularities are compatible with the present day observations. This difficulty is called the degeneracy problem. We use the Akaike and Bayesian information criteria of model selection to overcome this degeneracy and to determine a model with such a set of parameters which gives the most preferred fit to the SNIa data. We consider seven representative scenarios, namely: the CDM models with the cosmological constant, with topological defect, with phantom field, with bounce, with bouncing phantom field, with brane and model with the linear dynamical equation of state parameter. Applying the model selection information criteria we show that AIC indicates the flat phantom model while BIC indicates both flat phantom and flat ΛCDM models. Finally we conclude that the number of essential parameters chosen by dark energy models which are compared with SNIa data is two.

I. INTRODUCTION

The present observations of the distant supernovae type Ia indicate that the Universe is presently accelerating [1, 2] due to the presence of some unknown form of energy violating the strong energy condition $\rho_X + 3p_X > 0$ where ρ_X and p_X are energy density and pressure of dark energy, respectively. While the different candidates for dark energy were proposed [3, 4] and confronted with observations [5, 6, 7, 8, 9, 10, 11], the cosmological constant Λ and phantom fields [12, 13, 14] violating the weak energy condition $\rho_X + p_X > 0$ are most popular. Whereas both the cosmological constant and phantoms fields, described by the barotropic equation of state $p_i = w_i\rho$ ($w_i \leq -1$), are negligible in the neighborhood of the initial singularity, they dominate late time evolution. In the case of the Λ domination we obtain the de Sitter state as a global attractor in the future and for phantom fields there are a big rip singularity. Both the singularities are generic features of their models. In the case of the big rip singularity at the some finite time the both scale factor and energy density are growing at infinity and the unexpected future singularity of density appears [15, 16]. Alternatively the universe evolved from the Planck region where quantum gravity effects would have dominate. In this context quantum cosmology is used as a first approximation to the quantum gravity expected effects. Recently the proposal of quantum loop cosmology offers possibility of avoiding an initial singularity [17, 18]. For completeness we also consider the brane model [19, 20, 21] and the models with dynamical dark energy satisfying the dynamical equation of state $p_X = w_X(z)\rho_X$, where $w_X(z)$ is linearized around $z = 0$ (the present epoch) [22, 23].

Let us consider four different prototypes of dynamical behavior which take place in the neighborhood of the initial and final singularities. They give rise the seven representative scenarios, namely, the CDM model with Λ (ΛCDM), the CDM model with topological defect (TDCDM), the phantom CDM model (PhCDM), the bouncing ΛCDM model (BΛCDM), the bouncing phantom CDM model (BPhCDM), the brane ΛCDM model in Randall Sundrum version (BrΛCDM), and the model with the dynamical equation of state parameter linearized around the present epoch (DEQS). They are completed in Table I, where the dependence of the Hubble function on redshift z is given together with a number of models parameters.

We consider the FRW dynamics in which dark energy is present. The acceleration of the Universe is due to the presence of dark energy for which the equation of state is $p_X = w_X\rho$ and $w_X = \text{const} < 0$ (quintessence) or $p_X = w_X(z)\rho$ and w_X is variable with the cosmological time scale factor or redshift z. Therefore the basic equation determining the evolution is

$$H^2 = \frac{\rho_{\text{eff}}}{3} - \frac{k}{a^2}. \tag{1}$$

where $\rho_{\text{eff}}(a)$ is effective energy density of noninteracting "fluids", $k = \pm 1, 0$ is the curvature index. Equation (1) can be presented in terms of density parameters

$$\frac{H^2}{H_0^2} = \Omega_{\text{eff}}(z) + \Omega_{k,0}(1 + z)^2 \tag{2}$$

case	name of model	$H(z)$	free parameters	d
0	Einstein-de Sitter	$H = H_0\sqrt{\Omega_{\mathrm{m},0}(1+z)^3 + \Omega_{k,0}(1+z)^2}$	$H_0, \Omega_{\mathrm{m},0}$	2
1	ΛCDM	$H = H_0\sqrt{\Omega_{\mathrm{m},0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_\Lambda}$	$H_0, \Omega_{\mathrm{m},0}, \Omega_\Lambda$	3
2	TDCDM	$H = H_0\sqrt{\Omega_{\mathrm{m},0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{T,0}(1+z)}$	$H_0, \Omega_{\mathrm{m},0}, \Omega_{T,0}$	3
3a	PhCDM, $w=-\frac{4}{3}$	$H = H_0\sqrt{\Omega_{\mathrm{m},0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{Ph,0}(1+z)^{3(1+w)}}$	$H_0, \Omega_{\mathrm{m},0}, \Omega_{Ph,0}$	3
3b	PhCDM, w – fitted		$H_0, \Omega_{\mathrm{m},0}, \Omega_{Ph,0}, w$	4
4a	BΛCDM, $n=6$	$H = H_0\sqrt{\Omega_{\mathrm{m},0}(1+z)^3 + \Omega_{k,0}(1+z)^2 - \Omega_{n,0}(1+z)^n + \Omega_\Lambda}$	$H_0, \Omega_{\mathrm{m},0}, \Omega_{n,0}, \Omega_\Lambda$	4
4b	BΛCDM, n – fitted		$H_0, \Omega_{\mathrm{m},0}, \Omega_{n,0}, \Omega_\Lambda, n$	5
5a	BPhCDM, $n=6$	$H = H_0\sqrt{\Omega_{\mathrm{m},0}(1+z)^3 + \Omega_{k,0}(1+z)^2 - \Omega_{n,0}(1+z)^n + \Omega_{Ph,0}(1+z)^{-1}}$	$H_0, \Omega_{\mathrm{m},0}, \Omega_{n,0}, \Omega_{Ph,0}$	4
5b	BPhCDM, n – fitted		$H_0, \Omega_{\mathrm{m},0}, \Omega_{n,0}, \Omega_{Ph,0}, n$	5
6	BrΛCDM, $n=6$	$H = H_0\sqrt{\Omega_{\mathrm{m},0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{BRA,0}(1+z)^n + \Omega_\Lambda}$	$H_0, \Omega_{\mathrm{m},0}, \Omega_{BRA,0}, \Omega_\Lambda$	4
7a	DEQS, $w_0 = -1$, $p_X = (w_0 + w_1 z)\rho_X$	$H = H_0\sqrt{\Omega_{\mathrm{m},0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{X,0}(1+z)^{3(w_0-w_1+1)}e^{3w_1 z}}$	$H_0, \Omega_{\mathrm{m},0}, \Omega_{X,0}, w_1$	4
7b	DEQS, $w_0 =$ fitted, $p_X = (w_0 + w_1 z)\rho_X$		$H_0, \Omega_{\mathrm{m},0}, \Omega_{X,0}, w_0, w_1$	5

TABLE I: The Hubble function versus redshift for the seven evolutional scenarios of the FRW models with dark energy.

where $\frac{a}{a_0} = \frac{1}{1+z}$, $\Omega_{\mathrm{eff}}(z) = \Omega_{\mathrm{m},0}(1+z)^3 + \Omega_{X,0}f(z)$ and $\Omega_{\mathrm{m},0}$ is the density parameter for the (baryonic and dark) matter scaling like a^{-3}. For $a = a_0$ (the present value of the scale factor) we obtain the constraint $\Omega_{\mathrm{eff},0} + \Omega_{k,0} = 1$.

We assumed that energy density satisfies the conservation condition

$$\dot{\rho}_i = -3H(\rho_i + p_i), \tag{3}$$

for each component of the fluid $\rho_{\mathrm{eff}} = \Sigma\rho_i$. Then from eq. (2) we obtain the constraint relation $\Sigma_i\Omega_{i,0} + \Omega_{k,0} = 1$. In Table I we denote the numbers of parameters by d and also presented the names of the free parameters in the models. Please noted that for flat model, $\Omega_{k,0} = 0$ the number of the models parameters is equal $d-1$.

II.   DISTANT SUPERNOVAE AS COSMOLOGICAL PROBES OF DARK ENERGY

Distant type Ia supernovae surveys allowed us to find that the present Universe is accelerating [1, 2]. Every year new SNIa enlarge available data sets with more distant objects and lower systematics errors. Riess et al. [24] compiled the latest samples which become the standard data sets of SNIa. One of them, the restricted "Gold" sample of 157 SNIa, is used in our analysis.

For the distant SNIa one can directly observe their the apparent magnitude m and redshift z. Because the absolute magnitude $\mathcal{M}$ of the supernovae is related to its absolute luminosity L, then the relation between the luminosity distance d_L and observed m and absolute magnitude M has the following form

$$m - M = 5\log_{10} d_L + 25 \tag{4}$$

Instead of d_L, the dimensionless parameter D_L

$$D_L = H_0 d_L \tag{5}$$

is usually used and then eq. (4) changes to

$$\mu \equiv m - M = 5\log_{10} D_L + \mathcal{M} \tag{6}$$

where

$$\mathcal{M} = -5\log_{10} H_0 + 25. \tag{7}$$

We know the absolute magnitude of SNIa from the light curve. The luminosity distance of supernova can be obtain as the function of redshift

$$d_L(z) = (1+z)\frac{c}{H_0}\frac{1}{\sqrt{|\Omega_{k,0}|}}\mathcal{F}\left(H_0\sqrt{|\Omega_{k,0}|}\int_0^z \frac{dz'}{H(z')}\right) \tag{8}$$

where $\Omega_{k,0} = -\frac{k}{H_0^2}$ and

$$
\begin{aligned}
\mathcal{F}(x) &= \sinh(x) & \text{for} \quad k < 0 \\
\mathcal{F}(x) &= x & \text{for} \quad k = 0 \\
\mathcal{F}(x) &= \sin(x) & \text{for} \quad k > 0
\end{aligned}
\tag{9}
$$

Finally it is possible to probe dark energy which constitutes the main contribution to the matter content. It is assumed that supernovae measurements come with uncorrelated Gaussian errors and in this case the likelihood function $\mathcal{L}$ can be determined from the chi-square statistic $\mathcal{L} \propto \exp(-\chi^2/2)$ where

$$\chi^2 = \sum_i \frac{(\mu_i^{\mathrm{theor}} - \mu_i^{\mathrm{obs}})^2}{\sigma_i^2}, \tag{10}$$

while the probability density function of cosmological parameters is derived from Bayes' theorem [1]. Therefore, we can perform the estimation of model parameters using the minimization procedure, based on the likelihood function. Especially we can also derive a one-dimensional probability distribution functions (PDFs) for some model parameters.

For deeper statistical analysis we use the Akaike [25] and Bayesian information criteria [26]. It is crucial for our investigations to ask how well different theoretical models fits the same data set [27]. Because from the theoretical

TABLE II: The values of AIC and BIC for distinguished models (Table I) both for flat and non-flat model.

case	AIC ($\Omega_{k,0} = 0$)	AIC ($\Omega_{k,0} \neq 0$)	BIC ($\Omega_{k,0} = 0$)	BIC ($\Omega_{k,0} \neq 0$)
0	325.5	194.4	328.6	200.5
1	179.9	179.9	186.0	189.0
2	183.2	180.1	189.4	194.4
3a	178.0	179.3	184.1	188.5
3b	178.5	179.7	187.7	191.9
4a	181.9	181.6	191.1	193.8
4b	183.9	183.6	196.2	198.8
5a	180.0	181.3	189.2	193.5
5b	182.0	183.3	194.4	198.5
6	180.3	181.9	189.4	194.1
7a	179.8	181.6	188.9	193.8
7b	180.5	182.0	192.7	197.3
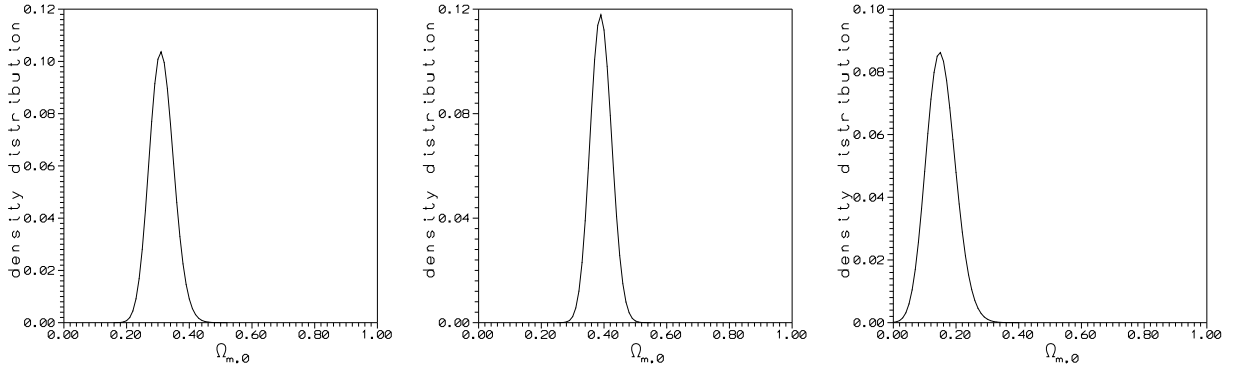
FIG. 1: The one-dimensional probability density distributions (PDFs) for $\Omega_{m,0}$, obtained for (from left to right) the ΛCDM, PhCDM, TDCDM models, respectively.

investigation arising a large number of candidates for dark energy description it is necessary to select the essential model parameters from the considered set of parameters of the most general model.

The Akaike information criterion (AIC) is defined in the following way [25]

$$\mathrm{AIC} = -2\ln\mathcal{L} + 2d \tag{11}$$

where $\mathcal{L}$ is the maximum likelihood and d is the number of the model parameters. The best model with a parameter set providing the preferred fit to the data is that minimizes the AIC.

The Bayesian information criterion (BIC) introduced by Schwarz [26] is defined as

$$\mathrm{BIC} = -2\ln\mathcal{L} + d\ln N \tag{12}$$

where N is the number of data points used in the fit.

The effectiveness of using this criteria in the current cosmological applications has been recently demonstrated by Liddle [27]. Taking CMB WMAP data [28] and applying the information criteria, Liddle [27] found the number of essential cosmological parameters to be five. Moreover he obtained the important conclusion that spatially-flat models are statistically preferred to close models as it was indicated by the CMB WMAP analysis (their best-fit value is $\Omega_{\mathrm{tot},0} \equiv \Sigma_i \Omega_{i,0} = 1.02 \pm 0.02$ at 1σ level).

Note that both information criteria have no absolute sense and only the relative value between differential models are physically interesting. For the BIC a difference of 2 is treated as a positive evidence (6 as a strong evidence) against the model with larger value of BIC [27, 29, 30].

The results of calculation AIC and BIC in the context of dark energy models are presented in Table II and Table III. In Table II we show results for all models considered for both flat and non flat cases without any assumed extra priors. In the general case the number of essential parameters in the cosmological models with dark energy is in principal two, i.e., $(H_0, \Omega_{m,0})$. It mean that flat model is favored in the light of the information criteria. We can
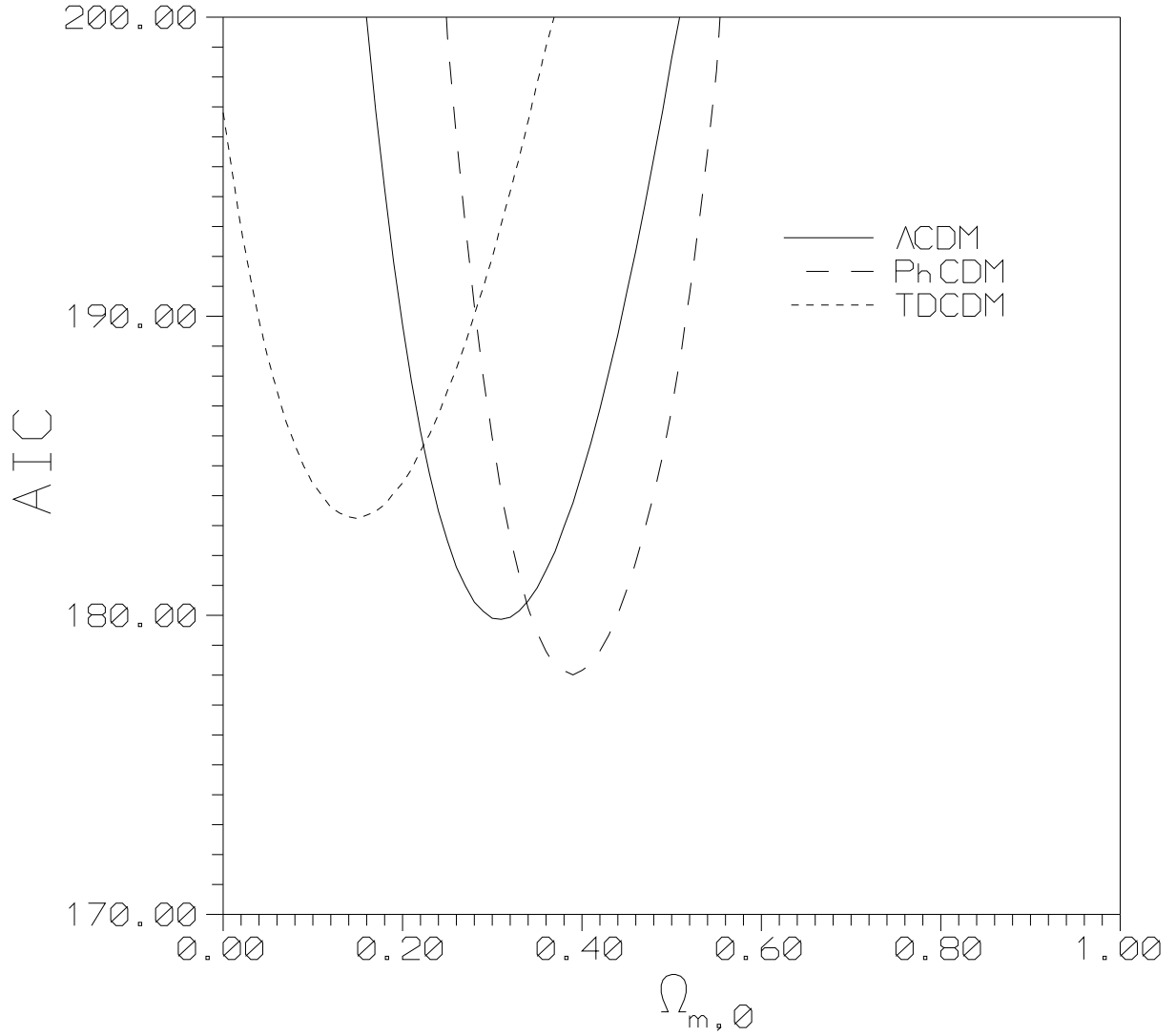
FIG. 2: The value of AIC in respect to fixed value $\Omega_{m,0}$ for three flat models (with topological defect, with cosmological constant and with phantom) with only one parameter H_0 estimated.

TABLE III: The values of AIC and BIC for distinguished models (Table I), with priors $\Omega_{m,0} = 0.3$ both for flat and non-flat model.

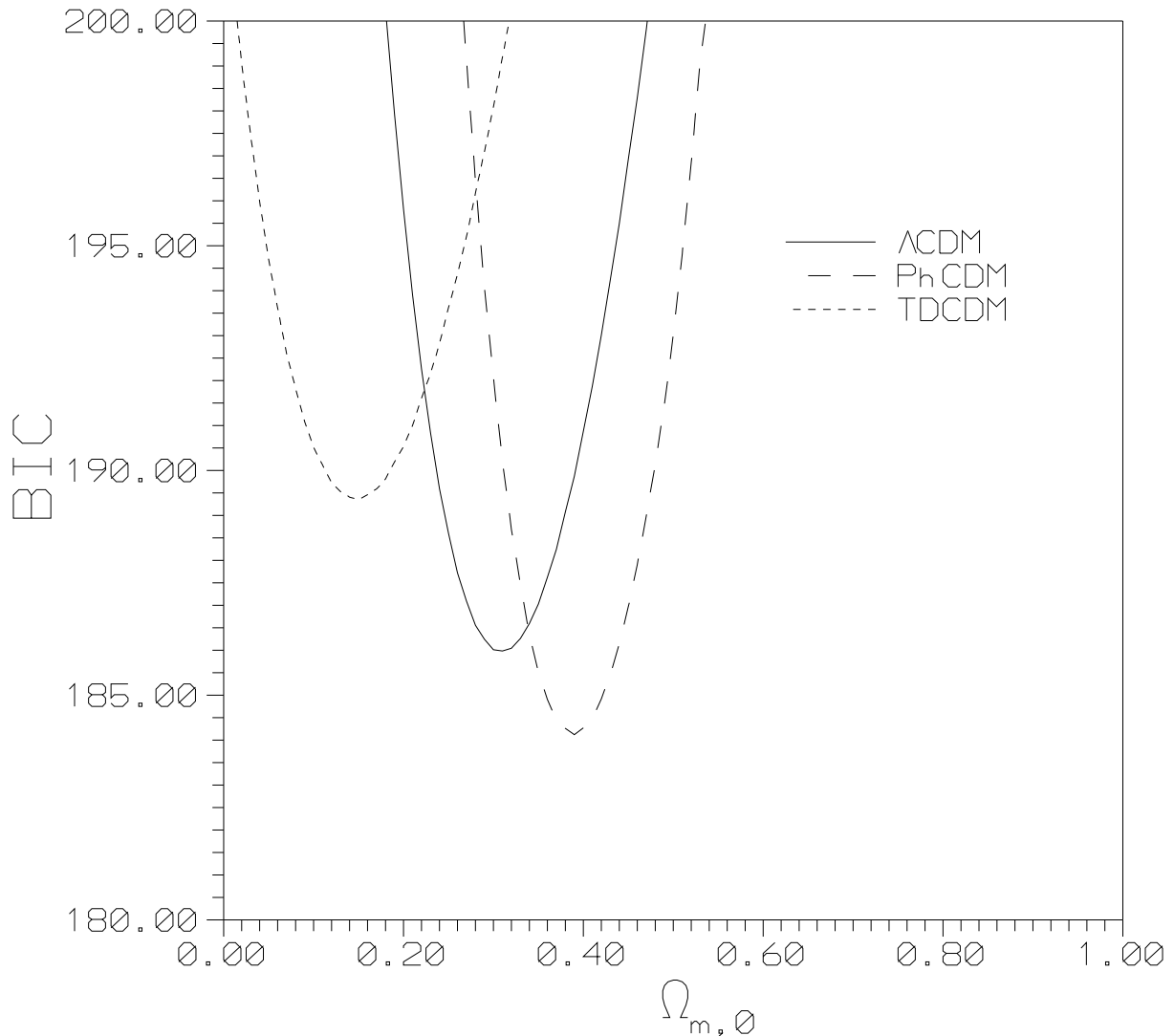| case	AIC ($\Omega_{k,0} = 0$)	AIC ($\Omega_{k,0} \neq 0$)	BIC ($\Omega_{k,0} = 0$)	BIC ($\Omega_{k,0} \neq 0$)
0	—	216.9	—	220.0
1	177.9	179.9	181.0	186.0
2	190.0	178.8	193.0	184.9
3a	183.9	179.6	187.0	186.7
3b	179.9	178.2	186.0	187.4
4a	179.9	181.9	186.0	191.0
4b	181.9	183.9	191.1	196.1
5a	185.9	181.6	192.0	190.7
5b	187.9	183.6	197.1	195.8
6	179.5	180.1	185.6	189.3
7a	179.7	179.6	185.8	188.7
7b	179.2	180.2	188.4	192.4

FIG. 3: The value of BIC in respect to fixed value $\Omega_{m,0}$ for three flat models (with topological defect, with the cosmological constant and with phantom) with only one parameter H_0 estimated.

observe two rival models which minimizes the AIC and BIC. They are the ΛCDM model and phantom CDM (PhCDM) models. One can observe that both BIC and AIC values assume greater values for phantom models. This evidence can be regarded as a positive evidence in favor of the PhCDM model.

We can derived one-dimensional probability distribution functions (PDFs) for the models parameters. In Fig. 1 we present PDFs for $\Omega_{m,0}$ for all three flat models with two free parameters, i.e., for the flat ΛCDM, TDCDM and PhCDM models. There is a significant difference between predictions of these models. The ΛCDM model prefers a universe with $\Omega_{m,0}$ close to 0.3, the PhCDM model favors a high density universe while the TDCDM model favors a low density universe. In Fig. 2 and Fig. 3 we present values of the AIC and BIC for these models. If $\Omega_{m,0} < 0.22$ then the information criteria favor the TDCDM model. For $\Omega_{m,0} \in (0.22, 0.34)$, the ΛCDM is favored while for $\Omega_{m,0} > 0.34$ the PhCDM model is preferred.

The similar analysis with the use of the information criteria is done in the case of the assumed prior $\Omega_{m,0} = 0.3$ [31] (Table III). In this case models with Λ is preferred over the models with phantoms, that is in contrary to the results obtained in the previous case of no priors for $\Omega_{m,0}$. It clearly shows that more precise measurements of $\Omega_{m,0}$ will give us the possibility to discriminate between the ΛCDM and PhCDM models.

III. CONCLUSION

The main goal of this letter is to decide which model with dark energy is distinguish by statistical analysis of SNIa data. To do this we use the Akaike and Bayesian information criteria. The former criterion weighs in favor of the flat phantom model while the latter distinguishes the flat phantom and ΛCDM models. Assuming the prior $\Omega_{\mathrm{m},0} = 0.3$ both AIC and BIC criteria weighs in favor of the flat ΛCDM model.

It was considered the seven different models containing dark energy and dust matter (baryonic and dark matter). Our main result is that spatially flat phantom model is favored to ΛCDM model by information criteria of AIC and BIC. The further conclusions are the following.

- The minimal number of essential parameters in the cosmological models with dark energy is in principal two, i.e., $(H_0, \Omega_{\mathrm{m},0})$. The list of essential parameters may be longer, because some of them are not convincingly measured with present data, like the parameter w_1.

- The curvature density parameter does not belong to the class of essential parameters when all the rest parameters are without any priors (with no fixed $\Omega_{\mathrm{m},0}$). At this point our result coincides with analogous result obtained by Liddle who found it basing on other observations.

- If we consider models in which all model parameters are fitted then the PhCDM model with double initial and final singularities is distinguished.

- When we consider the prior on $\Omega_{\mathrm{m},0}$, then the model with two-dimensional topological defect has $\Omega_{\mathrm{m},0} < 0.22$, the ΛCDM has $\Omega_{\mathrm{m},0} \in (0.22; 0.34)$, and the phantom model $\Omega_{\mathrm{m},0} > 0.34$.

To make the ultimate decision which model describes our Universe it is necessary to obtain the precise value of $\Omega_{\mathrm{m},0}$ from independent observations.

One can also draw from our analysis some kind of philosophical conclusion. Basing on simple and objective information criterion finally we obtain that SNIa data favored the models with the initial (big bang) and final (big rip) singularities. The big rip singularities are represents generic final future state of universe dominating by phantom dark energy. The bounce can be caused by high energy quantity modifications to FRW equations which make the cosmology nonsingular [32]. If the energy density is so large then quantum gravity corrections are important at both the big bang and big rip. It is interesting that classical theory reveals itself boundness, i.e., classical singularities which can be distinguished by the information criteria. The account of quantum effects allows to avoid not only the initial singularity [17, 18] but also escape from the future singularity [33, 34, 35].

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